Two-Phase Flow: The Concept of Similarity and Equivalent Transport Properties of Fluids

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Multiphase fluid transport phenomena are frequently met in industry and have been subjected to considerable study. Fortunately, most of the systems encountered involve only two phases—either solid-fluid or gas-liquid. The general solid-fluid systems have been studied extensively with a reasonable measure of ease and success because the solid phase is usually discontinuous and dispersed in the continuous fluid phase such as to give more or less homogeneous flow. By contrast, the complexity of the gas-liquid system has been well recognized for some time. The particular problems of this system are in no small measure attributable to the number of different flow characteristics which can be achieved by varying the flow rates of the two phases.

Undoubtedly, the most practical solution to problems in the gas-liquid two-phase system lie with the empirical approach. One of these problems is the pressure drop or the friction factor of the system. Dukler et al. (1964) proposed the use of dimensionless numbers which were derived through the concept of similarity and termed two-phase Reynolds number and two-phase Euler number. The analysis has proved the following results:

$$N_{ReTP} = l\overline{V}_m \left[\frac{\rho_L \overline{R}_L (\overline{V}_L/\overline{V}_m)^2 + \rho_G \overline{R}_G (\overline{V}_L/\overline{V}_m)^2 C_1}{\mu_L \overline{R}_L (\overline{V}_L/\overline{V}_m) + \mu_G \overline{R}_G (\overline{V}_L/\overline{V}_m) C_2} \right] \quad (1)$$

$$N_{EuTP} = 2f = \frac{\partial \overline{P}/\partial z}{\overline{V}_m^2/gl} \left[\frac{1}{\rho_L \overline{R}_L [\overline{V}_L/\overline{V}_m]^2 + \rho_G \overline{R}_G [\overline{V}_G/\overline{V}_m]^2 C_1} \right] \quad (2)$$

where

$$\overline{V}_m = \frac{Q_L + Q_G}{A} \Bigg|_{\text{at inlet}} \quad (3)$$

$$C_1 = \frac{R_G}{\overline{R}_G} \frac{\overline{R}_L}{R_L} \left(\frac{\overline{V}_L}{\overline{V}_G} \right)^2 \frac{V_G}{V_L} \frac{dV_G/dz}{dV_L/dz} \quad (4)$$

$$C_2 = \frac{R_G}{\overline{R}_G} \frac{\overline{R}_L}{R_L} \frac{\overline{V}_L}{\overline{V}_G} \frac{d^2V_G/dn^2}{d^2V_L/dn^2} \quad (5)$$

To avoid the complexity of these expressions, Dukler et al. divided the problem into four cases and assumed the following values of C_1 and C_2:

Case 1, Homogeneous flow, no slip, $C_1 = C_2 = 1.0$

Case 2, Slip takes place $C_1 = C_2 = 1.0$

Case 3, R_G small, $C_1 = C_2 = 0.0$

Case 4, $C_1 = C_2 = \overline{V}_L/\overline{V}_G$

Except for Case 1, these assumptions were shown to be arbitrary. The validity of the value 1.0 for C_1 and C_2, regardless of flow type, was proved experimentally by Dukler et al. (1964) themselves and also by Degance and Atherton (1970).

The purpose of this work is to use the Dukler analysis on a symmetrical system to show that the constants C_1 and C_2 in Equations (1) and (2) are in fact equal to 1.0.

Symmetrical system. In studying the transport phenomena of fluids flowing in a conduit, the two main areas of concern are the geometry of the conduit and the dynamics of the flowing fluids. If the conduit has a certain number of degrees of symmetry and so does the dynamics of the flowing fluids and if one (or more) degree of symmetry of both coincides, the system is defined as a symmetrical system. That is a system in which there are always at least (in the case of least symmetry) another elementary control volume which has the same geometry and the same magnitudes of forces and velocities as the one under study. The application of this definition will become clearer in the later section.

TWO-PHASE REYNOLDS AND EULER NUMBERS

In this section the analysis will be shown mainly for the two-phase Reynolds number; the Euler number can then be obtained in a similar manner.

In two-phase—phase I and phase II—flow, the derivation of the Reynolds number involves the comparison of the ratio inertial force/viscous force between two similar points q_A and q_B, say, of two similar systems A and B. As it has been shown (Dukler et al., 1964), the following expression can be obtained:

$$\frac{F_i}{F_v} = \left[\frac{\rho_I R_I V_I \dfrac{\partial V_I}{\partial z} + \rho_{II} R_{II} V_{II} \dfrac{\partial V_{II}}{\partial z}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II} \dfrac{\partial^2 V_{II}}{\partial n^2}} \right]_A = \left[\frac{\rho_I R_I V_I \dfrac{\partial V_I}{\partial z} + \rho_{II} R_{II} V_{II} \dfrac{\partial V_{II}}{\partial z}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II} \dfrac{\partial^2 V_{II}}{\partial n^2}} \right]_B \quad (6)$$

which consists of local quantities associated with a volume dv at the points q. The problem now is to replace these local quantities by their corresponding average quantities. Dukler et al. have used Shames' argument (1962) regarding the relationship between local quantities and average quantities to obtain Equation (1). However to show that C_1 and C_2 are equal to 1.0, it is necessary to redevelop the similarity analysis to a point where its application on a symmetrical system allows further simplification.

In Equation (6), by bringing the term $V_I \partial V_I/\partial z$ out of the bracket for both systems and using Shames' argument, it becomes

$$\frac{\overline{V}^2_{I,A}}{l_A}\left[\frac{\rho_I R_I + \rho_{II} R_{II}\dfrac{V_{II}\partial V_{II}/\partial z}{V_I \partial V_I/\partial z}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_A = \frac{\overline{V}^2_{I,B}}{l_B}\left[\frac{\rho_I R_I + \rho_{II} R_{II}\dfrac{V_{II}\partial V_{II}/\partial z}{V_I \partial V_I/\partial z}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_B \quad (7)$$

Similarly from Equation (6), if instead of the term $V_I \partial V_I/\partial z$, the term $V_{II}\partial V_{II}/\partial z$ is brought out of the bracket for both systems, the following expression should result:

$$\frac{\overline{V}^2_{II,A}}{l_A}\left[\frac{\rho_I R_I \dfrac{V_I\partial V_I/\partial z}{V_{II} \partial V_{II}/\partial z} + \rho_{II} R_{II}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_A = \frac{\overline{V}^2_{II,B}}{l_B}\left[\frac{\rho_I R_I \dfrac{V_I\partial V_I/\partial z}{V_{II} \partial V_{II}/\partial z} + \rho_{II} R_{II}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_B \quad (8)$$

It is noted that the derivation of Equations (7) and (8) is merely a reworking of Dukler's analysis. The two equations are, in general, not the same. Consider a symmetrical system which has at least one degree of symmetry in its geometry and at least one degree of symmetry in its dynamics. A diagram of the systems in the case of least symmetry is shown in Figure 1.

Ψ_1 and Ψ_2 represent the dynamical characteristics of the two control volumes dv_1 and dv_2. The superposition of the two symmetries will result with $dv_1 = dv_2$ and

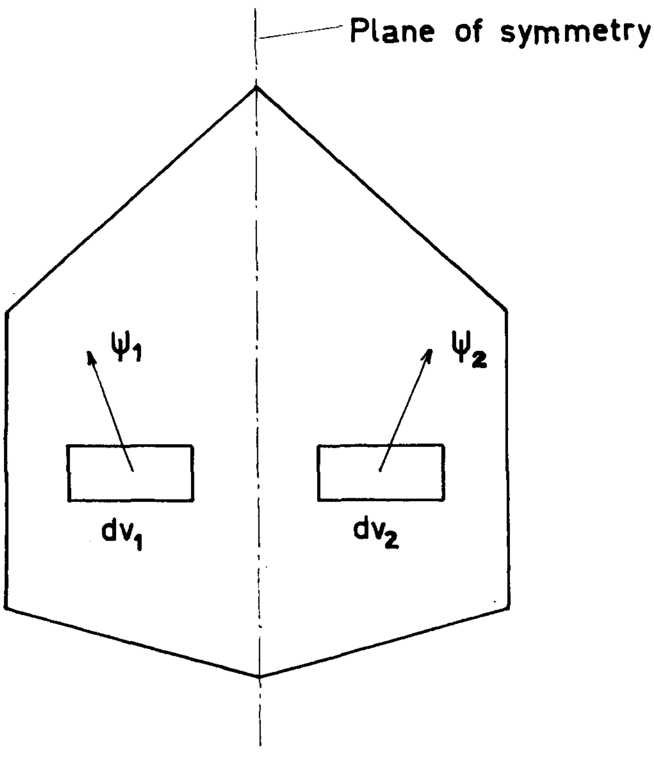

Fig. 1. The cross section of a symmetrical system.

$|\Psi_1| = |\Psi_2|$. In two similar systems A and B, the Equation (6) can be established for both pairs $dv_{1,A}$, $dv_{1,B}$ and $dv_{2,A}$, $dv_{2,B}$ regardless of the symmetrical characteristics. By assigning the results of the similarity analysis, namely Equation (7), to the pair $dv_{1,A}$ and $dv_{1,B}$ and Equation (8) to the pair $dv_{2,A}$ and $dv_{2,B}$, that is,

$$\text{Equation (7)}: \ \Psi_{1,A} = \Psi_{1,B}$$

$$\text{Equation (8)}: \ \Psi_{2,A} = \Psi_{2,B}$$

and if A and B are both symmetrical, then

$$|\Psi_{1,A}| = |\Psi_{2,A}| \quad (9)$$

and

$$|\Psi_{1,B}| = |\Psi_{2,B}| \quad (10)$$

that is, for system A, the expressions in the left hand side of Equation (7) and (8) are equal or

$$\frac{\overline{V}^2_{I,A}\left(\rho_I R_I + \rho_{II} R_{II}\dfrac{V_{II}\partial V_{II}/\partial z}{V_I\partial V_I/\partial z}\right)_A}{\overline{V}^2_{II,A}\left(\rho_I R_I \cdot \dfrac{V_I\partial V_I/\partial z}{V_{II}\partial V_{II}/\partial z} + \rho_{II} R_{II}\right)_A} = 1 \quad (11)$$

Multiply both sides of Equation (11) by $(V_I\partial V_I/\partial z)_A/(V_{II}\partial V_{II}/\partial z)_A$; the terms in brackets in denominator and numerator of the left hand side will cancel each other out, giving

$$\frac{\overline{V}^2_{I,A}}{\overline{V}^2_{II,A}} = \left[\frac{V_I\partial V_I/\partial z}{V_{II}\partial V_{II}/\partial z}\right]_A \quad (12)$$

Similarly when system B is considered,

$$\frac{\overline{V}^2_{I,B}}{\overline{V}^2_{II,B}} = \left[\frac{V_I\partial V_I/\partial z}{V_{II}\partial V_{II}/\partial z}\right]_B \quad (13)$$

Substitute into (8) and bringing the terms $\overline{V}^2_{II}$ into the brackets, results in

$$\frac{1}{l_A}\left[\frac{\rho_I R_I \overline{V}^2_I + \rho_{II} R_{II} \overline{V}^2_{II}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_A = \frac{1}{l_B}\left[\frac{\rho_I R_I \overline{V}^2_I + \rho_{II} R_{II} \overline{V}^2_{II}}{\mu_I R_I \dfrac{\partial^2 V_I}{\partial n^2} + \mu_{II} R_{II}\dfrac{\partial^2 V_{II}}{\partial n^2}}\right]_B \quad (14)$$

The same procedure can now be carried out to eliminate the terms $\partial^2 V/\partial n^2$ and the result is

$$l_A\left[\frac{\rho_I R_I \overline{V}^2_I + \rho_{II} R_{II} \overline{V}^2_{II}}{\mu_I R_I \overline{V}_I + \mu_{II} R_{II} \overline{V}_{II}}\right]_A = l_B\left[\frac{\rho_I R_I \overline{V}^2_I + \rho_{II} R_{II} \overline{V}^2_{II}}{\mu_I R_I \overline{V}_I + \mu_{II} R_{II} \overline{V}_{II}}\right]_B \quad (15)$$

The physical properties of the phases (μ, ρ) are dependent mainly on the pressure and temperature. These do not change significantly across the section of the flow conduits, therefore the terms ρ_I, ρ_{II}, μ_I and μ_{II} for the two systems A and B in Equation (15) can be taken as the average quantities at the sections passing the points q_A and q_B under consideration. The magnitudes of the expressions in each side of the Equation (15) change with the remaining local variable R_I, say, from one pair of points q_A and q_B to another. However, the equality between the two systems always holds. This local volumetric fraction is dimensionless, that is, its magnitude is independent of dimensions of the conduits, force scales, and velocity scales of the two systems A and B, and is only dependent on the pairs q_A and q_B. Therefore, as long as these pairs retain the similarity characteristic, the use of the average value over the cross section of the conduits for R_I and R_{II} will not affect the equality of the equation:

$$l_A = \left[\frac{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}}{\mu_I \overline{R}_I \overline{V}_I + \mu_{II} \overline{R}_{II} \overline{V}_{II}} \right]_A = l_B \left[\frac{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}}{\mu_I \overline{R}_I \overline{V}_I + \mu_{II} \overline{R}_{II} \overline{V}_{II}} \right]_B \quad (16)$$

Similarly the following expression can be obtained for Euler number:

$$l_A \left[\frac{\partial \overline{P}/\partial z}{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}} \right]_A = l_B \left[\frac{\partial \overline{P}/\partial z}{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}} \right]_B \quad (17)$$

In Equation (17), the rate of change of the local pressure in the z-direction is replaced by that of the average pressure at the section under study. This is justified by the fact that if this quantity differs from one point to another in the section it would result in a significant difference between local points at a section further downstream, a circumstance which is not possible in practice, hence

$$\partial P/\partial z = \overline{\partial P/\partial z} = \partial \overline{P}/\partial z$$

The Equations (16) and (17) are no longer the exact ratios Fi/Fv and Fp/Fi, but they do represent their characteristics which are termed Reynolds number and Euler number. For gas-liquid two-phase flow in a circular conduit, the characteristic length is the conduit diameter, D:

$$N_{ReTP} = D \frac{\rho_L \overline{R}_L \overline{V}^2{}_L + \rho_G \overline{R}_G \overline{V}^2{}_G}{\mu_L \overline{R}_L \overline{V}_L + \mu_G \overline{R}_G \overline{V}_G} \quad (18)$$

$$N_{EuTP} = D \frac{\partial \overline{P}/\partial z}{\rho_L \overline{R}_L \overline{V}^2{}_L + \rho_G \overline{R}_G \overline{V}^2{}_G} = 2f \quad (19)$$

It is clear now that Dukler's case IV where $C_1 = C_2 = \overline{V}_L/\overline{V}_G$ does not exist in a symmetrical system which is normally encountered.

EQUIVALENT TRANSPORT PROPERTIES OF TWO-PHASE FLOW

Equivalent transport properties of two phases flowing in a conduit can now be derived:

$$N_{ReTP} = D \frac{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}}{\mu_I \overline{R}_I \overline{V}_I + \mu_{II} \overline{R}_{II} \overline{V}_{II}} \text{ equivalent to } D \frac{\rho_{TP} \overline{V}_{TP}}{\mu_{TP}} \quad (20)$$

$$N_{EuTP} = D \frac{\partial \overline{P}/\partial z}{\rho_I \overline{R}_I \overline{V}^2{}_I + \rho_{II} \overline{R}_{II} \overline{V}^2{}_{II}} \text{ equivalent to } D \frac{\partial \overline{P}/\partial z}{\rho_{TP} \overline{V}^2{}_{TP}} \quad (21)$$

Total mass flow rate, $W_{TP} = W_I + W_{II}$ that is,

$$\rho_{TP} \overline{V}_{TP} = \rho_I \overline{V}_I \overline{R}_I + \rho_{II} \overline{V}_{II} \overline{R}_{II} \quad (22)$$

In Equation (22), the volumetric fraction $\overline{R}$ is used as the fraction of the conduit cross-sectional area through which the phase is flowing. This is correct since the limit of $\overline{R}$, as the length in the direction of flow tends to zero, is the area fraction.

The results of three equations with three unknowns, ρ_{TP}, μ_{TP}, $\overline{V}_{TP}$ are

$$\rho_{TP} = \frac{1}{\dfrac{x^2{}_{mI}}{\rho_I \overline{R}_I} + \dfrac{x^2{}_{mII}}{\rho_{II} \overline{R}_{II}}} \quad (23)$$

$$\mu_{TP} = \mu_I \frac{\overline{V}_I}{\overline{V}_{TP}} \overline{R}_I + \mu_{II} \frac{\overline{V}_{II}}{\overline{V}_{TP}} \overline{R}_{II} \quad (24)$$

$$\overline{V}_{TP} = x_{mI} \overline{V}_I + x_{mII} \overline{V}_{II} \quad (25)$$

where x_{mI} = mass flow rate fraction of phase I = $\dfrac{W_I}{W_I + W_{II}}$.

CONCLUSION

Dukler's assumption that the constants C_1 and C_2 are equal to 1.0 is shown analytically to be correct for a symmetrical system. Many workers have used this symmetrical characteristic of two-phase, gas-liquid flow in pipes to simplify their experimental work in such areas as liquid entrainment, liquid film thickness measurement in the region of annular flow, liquid hold up measurement using radiation, etc. . . . , and this work gives a theoretical base for this assumption of the symmetrical behavior in two-phase flow.

ACKNOWLEDGMENT

The authors wish to express appreciation of Dr. T. H. Johanessen's notes on this work.

NOTATION

C_1, C_2 = constants in Dukler's analysis, dimensionless
D = pipe inside diameter, L
f = friction factor [Equation (2)], dimensionless
F = local force quantity, MLT^{-2}
l = characteristic length of a system, L
n = length in the direction normal to the flow, L
N_{Re} = Reynolds number, dimensionless
N_{Eu} = Euler number, dimensionless
P = local pressure force, $ML^{-1}T^{-2}$
$\overline{P}$ = average pressure force over the cross section of pipe, $ML^{-1}T^{-2}$
q = study point in the system, dimensionless
Q = volumetric flow rate, L^3T^{-1}
R = local volumetric fraction, dimensionless
$\overline{R}$ = average volumetric fraction over the cross section of pipe, dimensionless
v = control volume under study, L^3
V = local velocity, LT^{-1}
$\overline{V}$ = average velocity over the cross section of pipe, LT^{-1}
$\overline{V}_m$ = Dukler's mean velocity of two phase, LT^{-1}
x_m = mass flow rate fraction, dimensionless
z = length in flow direction, L

Subscripts

A = System A
B = System B
G = gas phase
i = inertial

L = liquid phase
p = pressure
TP = two-phase flow
v = viscous
I = Phase I
II = Phase II
1 = for control volume 1
2 = for control volume 2

Greek Letters

Ψ = dynamical characteristic
μ = viscosity, $ML^{-1}T^{-1}$
ρ = density, ML^{-3}

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Manuscript received October 4, 1972 and accepted December 27, 1972.

Stability of a Laminar Jet of Viscous Liquid—Influence of Nozzle Shape

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For a long time there has been an interest in the many diverse applications of laminar liquid jets and, consequently, their stability. The purpose of this study is to single out nozzle shape, since it has received little systematic attention in the past. In particular, it is shown that data for zero length orifices can be interpreted in terms of the better known properties of long cylindrical nozzles. A possible extension to nozzles of arbitrary shape and length is discussed. The fluids considered are Newtonian (constant viscosity coefficient), and the influence of the ambient atmosphere is included.

GENERAL DISCUSSION

For the initial portion of the breakup curve for cylindrical nozzles (the segment AB in Figure 1), breakup length is proportional to velocity. This segment has been much studied in the past and can be considered to be well understood. Experimental results are generally in agreement with each other and in turn are in agreement with theory. Grant and Middleman (1966) and Meister and Scheele (1967) give summaries of previous work and the progress to date. Almost all experimental data are concerned with long cylindrical nozzles.

The cause of the peak in the breakup curve (point C in Figure 1) has been a problem for some time. It seems likely that there is not a single cause but actually two competing ones. The nondimensional parameters that determine the onset of each of these causes are discussed below.

The relative motion of the jet and the ambient gas produces a pressure increment on the jet surface that tends to amplify small disturbances of the surface. The parameter that controls this process is the Weber number based upon the ambient gas density We_a. Fenn and Middleman (1969) identified this parameter and experimentally found the critical value to be $We_a = 5.3$. The theory of Weber (1931), that includes this ambient influence, can be simplified to the point that calculations can be made easily without making restrictive assumptions (Phinney, 1973). It is found that for low jet viscosity the critical Weber number is largely independent of viscosity and is close to the value given by Fenn and Middleman (1969). In addition, the theory predicts the shape of the breakup curve up to and beyond the peak. These predictions agree reasonably well with experimental observations.

The above discussion is based upon the assumption that the exit flow has a constant disturbance level at the jet exit, independent of the jet velocity. It is found, however, that for long cylindrical nozzles the disturbance level increases with velocity after some threshold value defined by a critical Reynolds number $\hat{Re}$ (Phinney, 1972). This increase in disturbance level seems to be connected with some Tollmien-Schicting type of instability that ultimately leads to transition and turbulent flow in the pipe.

For any particular jet from a long nozzle, the peak in the breakup length-velocity curve will be due to one of the above causes. Which cause is determined in a particular case by which of the critical velocities is the lowest.

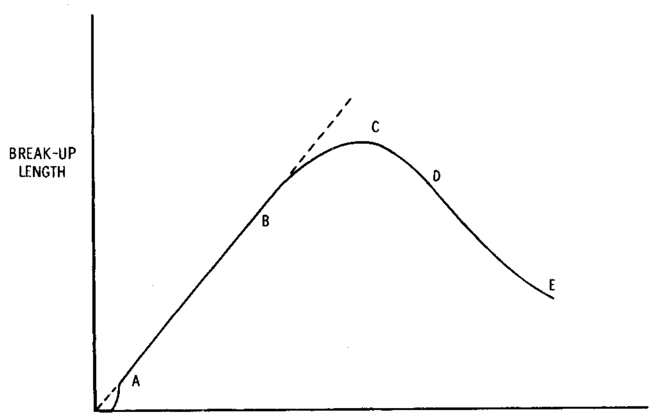

Fig. 1. Schematic of a breakup length-velocity curve.